Two-Dimensional Shape Optimization with Unstructured Meshes

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Motivation for Automatic Design

- Aerodynamic development typically “cut&try”
  - Slow (design time doing detailed design iterations)
  - Expensive
  - Relies on physical insight of designer for changes
- Automatic design to reduce time in detail design phase
  - Improved performance
  - Decreased costs

Aerodynamic Shape Optimization

- Large number of design variables necessary for complete aircraft
- Control theory - gradient requires only the solution of an adjoint system
- Gradient calculation independent of the number of design variables

Design Procedure

1. Solve the flow equations for **r**, u_1, u_2, u_3, p
2. Solve the adjoint equations for y subject to appropriate boundary conditions
3. Evaluate the gradient G
4. Project G into an allowable subspace that satisfies any geometric constraints
5. Update the shape based on the direction of steepest descent
6. Return to step 1 until convergence is reached

Euler Flow Solver/Design Code
SYN75

- 2D compressible inviscid fluid flow
- Finite volume
- Explicit multistage scheme of Jameson
- Multigrid time stepping scheme of Jameson
- Adjoint formulation for design problem
- General triangular mesh
- Equivalent to Galerkin finite element method

Governing Equations

Euler equations for flow of a compressible inviscid fluid in integral form:

$$\frac{\partial}{\partial t}\iiint_{\Omega} \mathbf{w}\, d\Omega + \iint_{\partial\Omega} \mathbf{F}\cdot d\mathbf{S} = 0$$

mass

$$\mathbf{w} = \rho$$

$$\mathbf{F} = (\rho u, \rho v)$$

momentum (for x)

$$\mathbf{w} = \rho u$$

$$\mathbf{F} = (\rho u^2 + p, \rho u v)$$

energy

$$\mathbf{w} = \rho E$$

$$\mathbf{F} = (\rho H u, \rho H v)$$

$$E = \frac{p}{(\gamma - 1)\rho} + \frac{1}{2}\left(u^2 + v^2\right)$$

Equation of State

Governing Equations

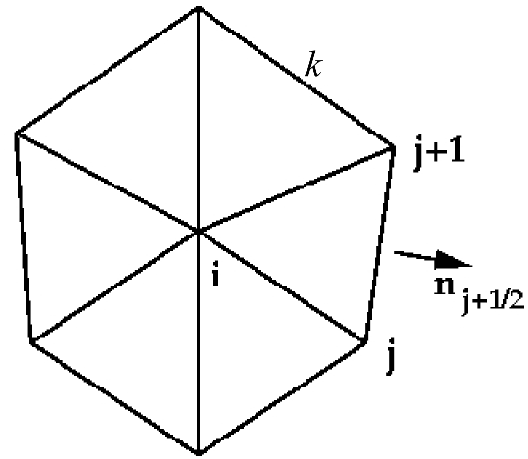
Previous equations are solved for each CV in computational domain

$$\frac{d}{dt}(\mathrm{V}_i)\mathbf{w}_i + \sum_k \mathrm{R}_k = 0$$

Flux contributions across interior faces cancel

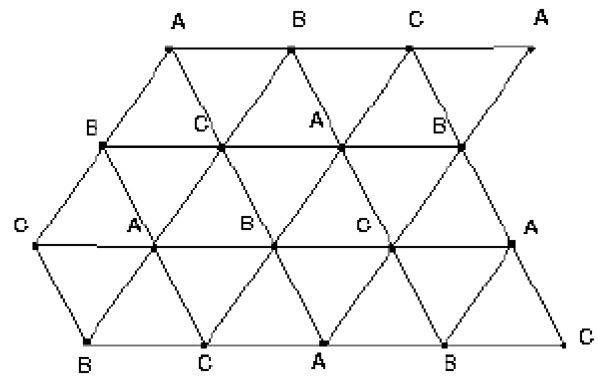
$$\frac{d}{dt}(\mathrm{V}_i)\mathbf{w}_i + \sum_k \mathbf{F}_k \cdot \mathbf{S}_k = 0$$

Control Volume for node i



Artificial Dissipation

Central differencing of the convective flux term allows oscillation of the solution due to even-odd decoupling



A,B,C constant but distinct

Add dissipation terms to the Euler fluxes

Integration In Time

Multistage Scheme

Jameson's Runge-Kutta method

for a m-stage scheme:

$$\mathbf{w}^{(0)} = \mathbf{w}^{n}$$

$$\mathbf{w}^{(k)} = \mathbf{w}^{(0)} - \alpha_k \Delta t \frac{1}{\mathrm{V}} R(\mathbf{w})^{k-1} k = 1,2,3,\ldots m$$

$$\mathbf{w}^{(n+1)} = \mathbf{w}^{m}$$

Convergence Acceleration

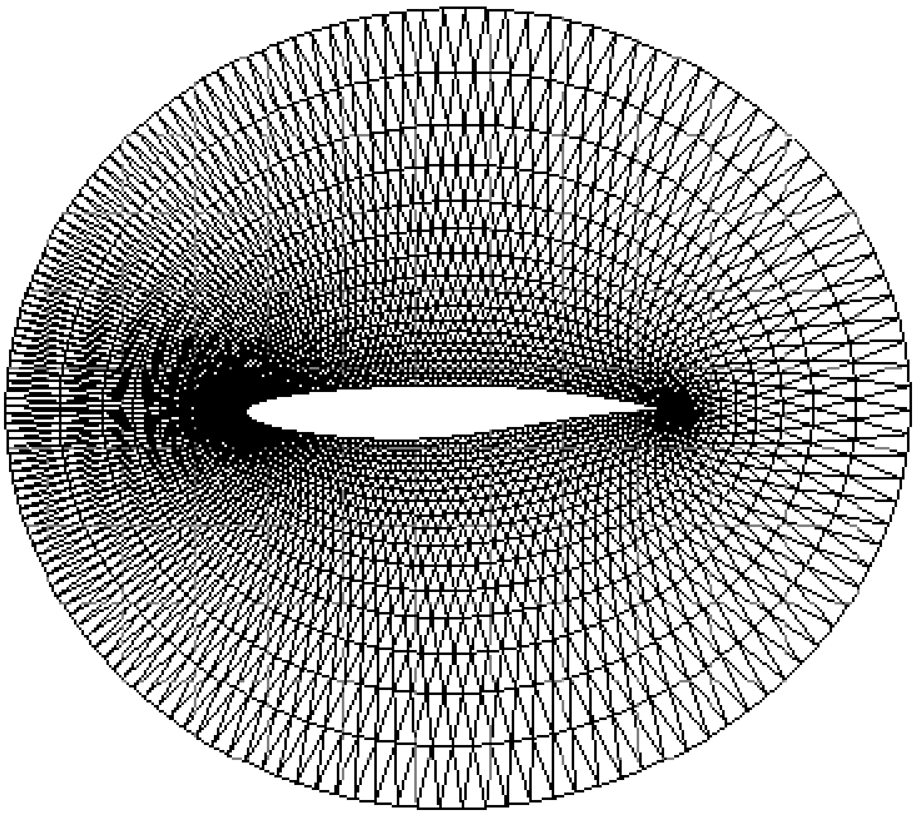
- Time step constraint for explicit schemes is too restrictive
- Convergence can be enhanced by
  - Local time stepping
  - Multigrid

Multigrid

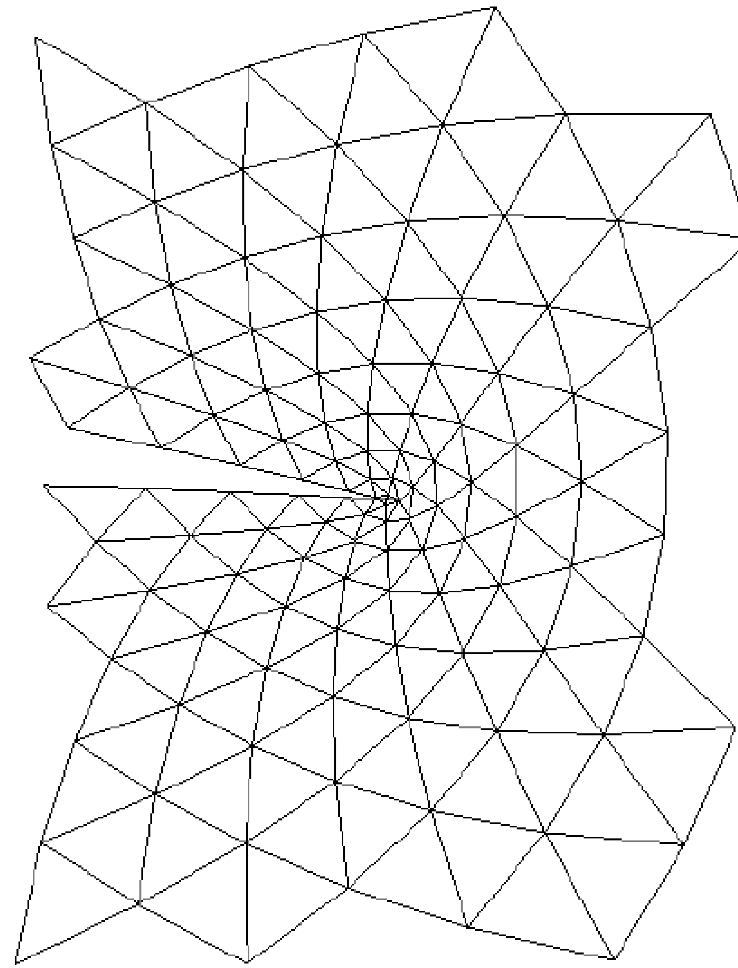
- Coarse meshes are used to estimate the correction to the residual calculated on the fine mesh
- For minimal extra computational cost and memory, convergence rates are increased by an order of magnitude

- First utilized for Euler equations on an unstructured grid by A. Jameson

Multigrid: Fine Grid

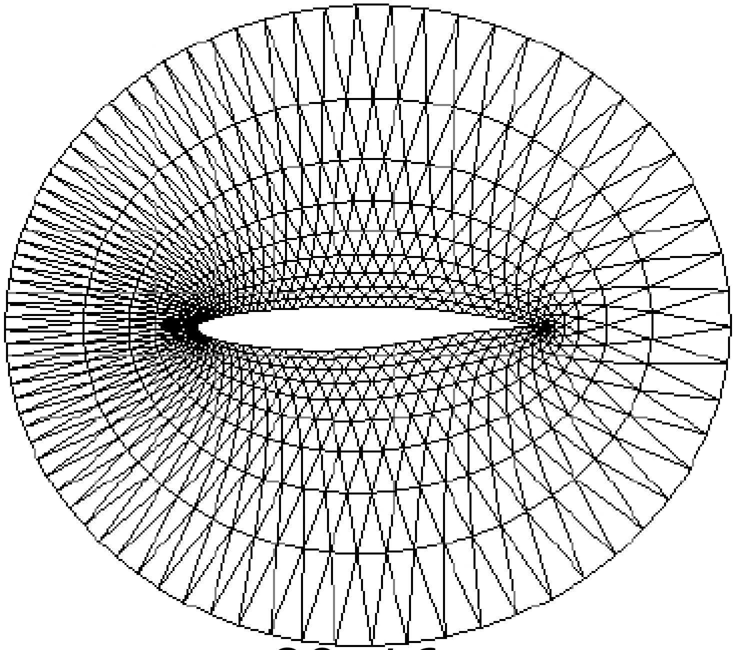


Fine mesh 160x32 points
(view of partial mesh)

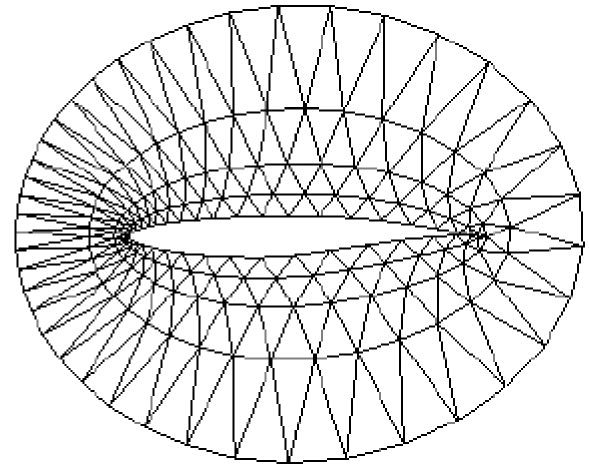


Trailing edge detail

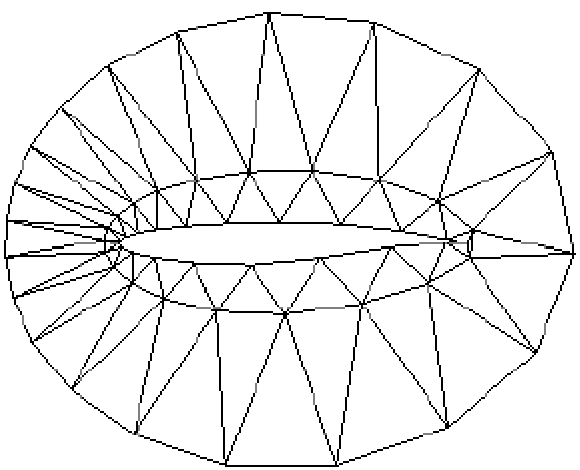
Multigrid: Coarse Grids



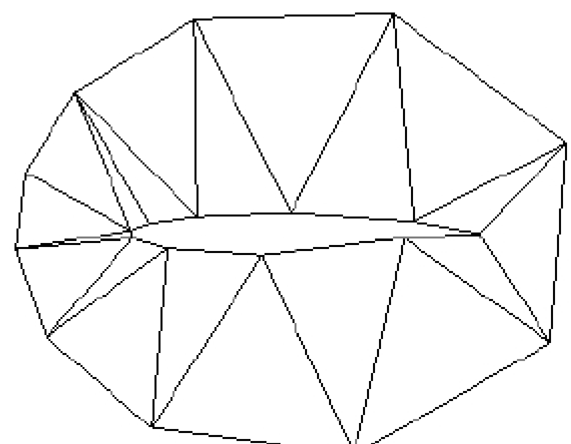
80x16



40x8



20x4



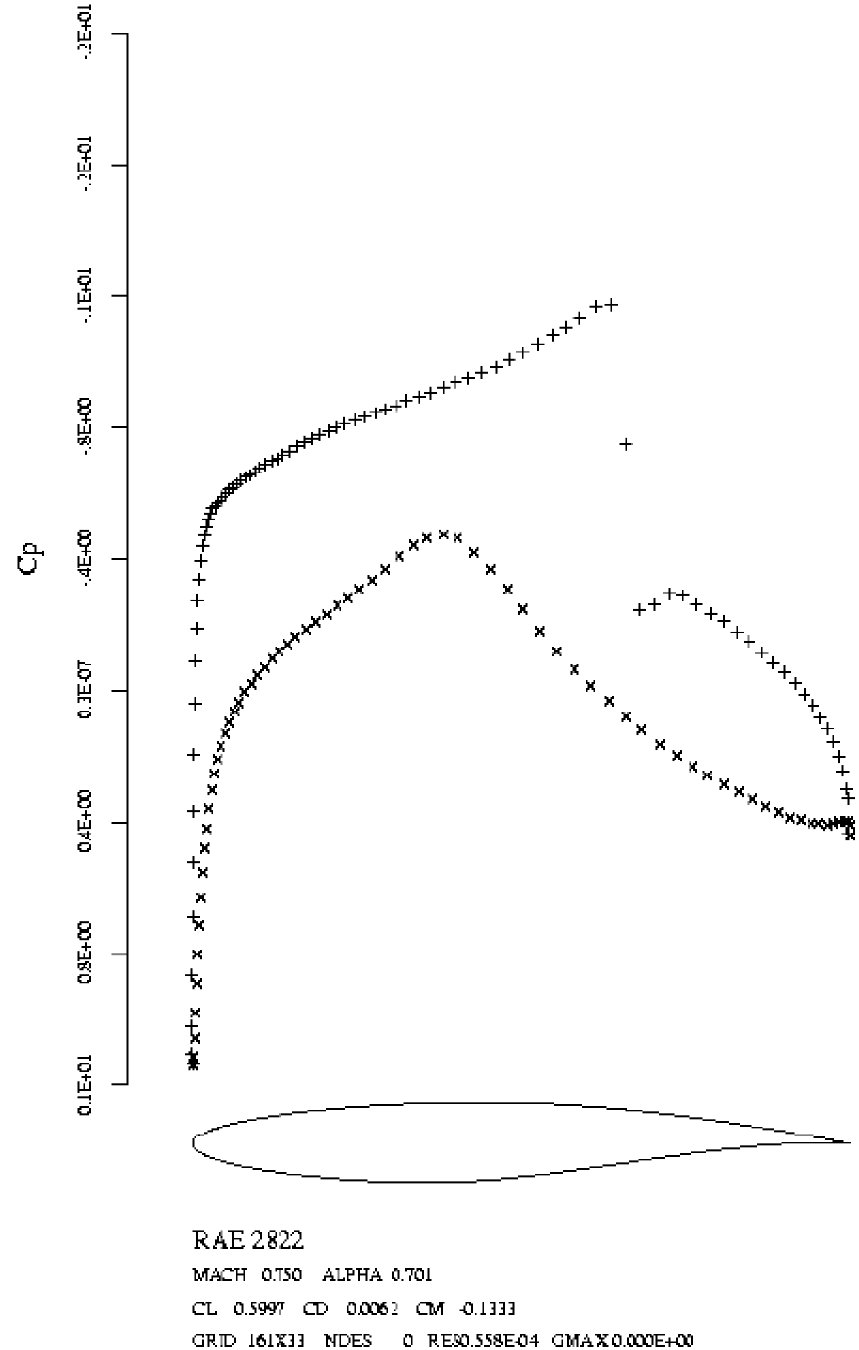
10x2

SYN75 Results

RAE drag minimization

Initial solution

C_D=0.0062

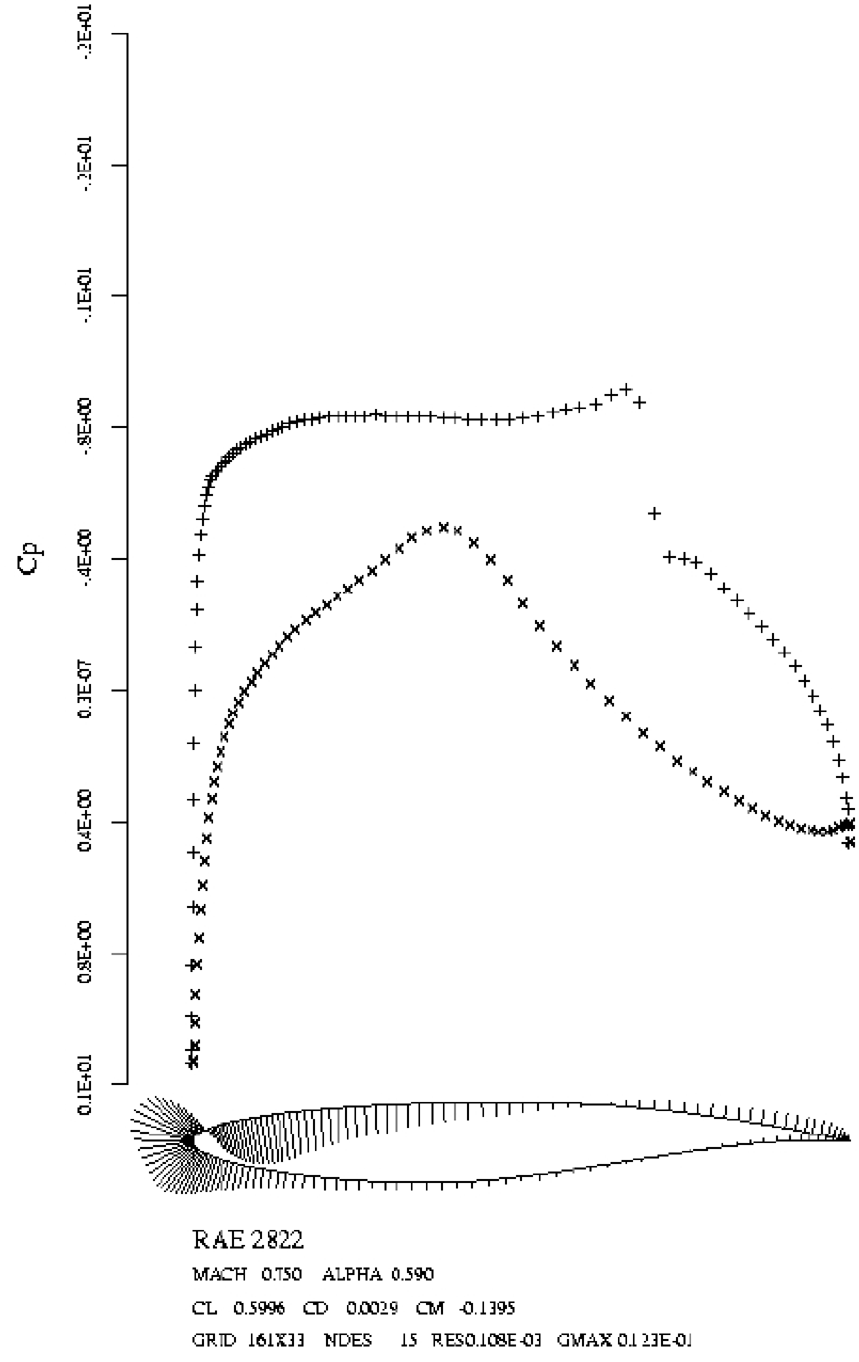


SYN75 Results

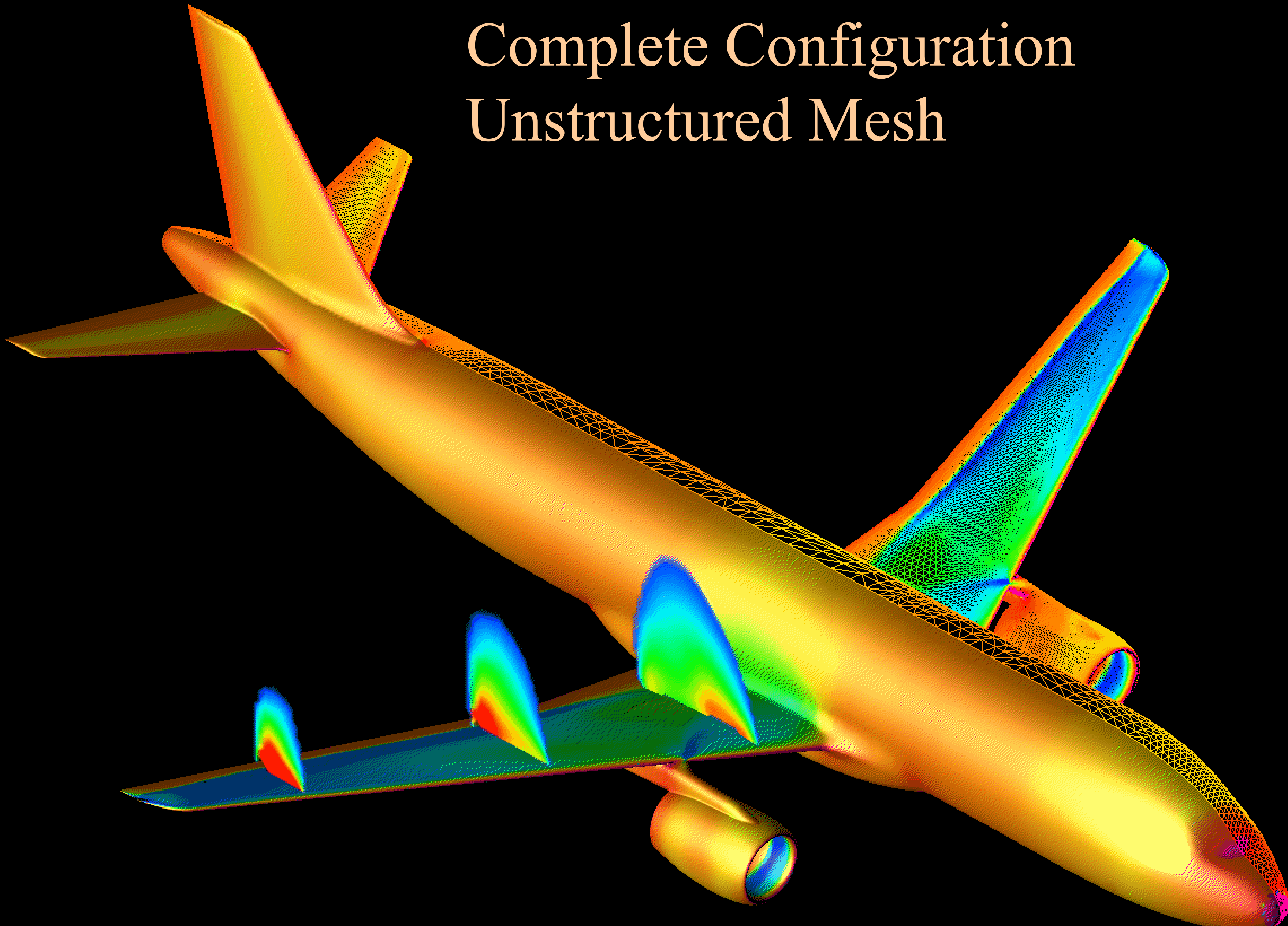
RAE drag minimization

15 design cycles

C_D=0.0029



Complete Configuration
Unstructured Mesh



Computational Requirements

- 2D - few design variables
  - Single processor
- 3D - large number of design variables
  - Serial computational time excessive
  - Parallel
    - Distribute work spatially: Domain Decomposition

Conclusion

- Adjoint formulation on unstructured mesh implemented
- Two-Dimensional unstructured mesh design tested

Work Plan

- Continue to develop 2D code
- Extend to 3D
  - Flow & adjoint solvers are in place (Jameson & Martinelli)
  - Write gradient formulation
- Shape modification
  - More general than point movement
  - Integration with CAD
- Pass Generals